

Inference Rules For Functional Dependencies

Inference Rules For Functional Dependencies -

Let S be the set of functional dependencies that are specified on relation schema R . Numerous other dependencies can be inferred or deduced from the functional dependencies in S .

Example :

Let $S = \{A \twoheadrightarrow B, B \twoheadrightarrow C\}$

We can infer the following functional dependency from S :

$A \twoheadrightarrow C$

Armstrong's Inference Rules -

Let A , B and C and D be arbitrary subsets of the set of attributes of the given relation R , and let AB be the union of A and B . Then,??

Reflexivity :

If B is subset of A , then $A \twoheadrightarrow B$

Augmentation :

If $A \twoheadrightarrow B$, then $AC \twoheadrightarrow BC$

Transitivity :

If $A \twoheadrightarrow B$ and $B \twoheadrightarrow C$, then $A \twoheadrightarrow C$.

Projectivity or Decomposition Rule :

If $A \twoheadrightarrow BC$, Then $A \twoheadrightarrow B$ and $A \twoheadrightarrow C$

Proof :

Step 1 : $A \twoheadrightarrow BC$ (GIVEN)

Step 2 : $BC \twoheadrightarrow B$ (Using Rule 1, since $B \twoheadrightarrow BC$)

Step 3 : $A \twoheadrightarrow B$ (Using Rule 3, on step 1 and step 2)

Union or Additive Rule :

If $A \twoheadrightarrow B$, and $A \twoheadrightarrow C$ Then $A \twoheadrightarrow BC$.

Proof :

Step 1 : $A \twoheadrightarrow B$ (GIVEN)

Step 2 : $A \twoheadrightarrow C$ (given)

Step 3 : $A \twoheadrightarrow AB$ (using Rule 2 on step 1, since $AA=A$)

Step 4 : $AB \twoheadrightarrow BC$ (using rule 2 on step 2)

Step 5 : $A \twoheadrightarrow BC$ (using rule 3 on step 3 and step 4)

Pseudo Transitive Rule :

If $A \twoheadrightarrow B$, $DB \twoheadrightarrow C$, then $DA \twoheadrightarrow C$

Proof :

Step 1 : $A \twoheadrightarrow B$ (Given)

Step 2 : $DB \twoheadrightarrow C$ (Given)

Step 3 : $DA \twoheadrightarrow DB$ (Rule 2 on step 1)

Step 4 : $DA \twoheadrightarrow C$ (Rule 3 on step 3 and step 2)

These are not commutative as well as associative.

i.e. if $X \twoheadrightarrow Y$ then

$Y \twoheadrightarrow X$ x (not possible)

Composition Rule :

If $A \twoheadrightarrow B$, and $C \twoheadrightarrow D$, then $AC \twoheadrightarrow BD$.

Self Determination Rule :

A \rightarrow A is a self determination rule.

Question 1:

Prove or disprove the following inference rules for functional dependencies.

Note: Read " \rightarrow " as implies

a. $\{X \rightarrow Y, Z \rightarrow W\} \rightarrow XZ \rightarrow YW$??

b. $\{X \rightarrow Y, XY \rightarrow Z\} \rightarrow X \rightarrow Z$

c. $\{XY \rightarrow Z, Y \rightarrow W\} \rightarrow XW \rightarrow Z$

Solution :

Method : Use Armstrong's Axioms or Attribute closure to prove or disprove.

a. $\{X \rightarrow Y, Z \rightarrow W\} \rightarrow XZ \rightarrow YW$??

$XZ \rightarrow XZ$

$XZ \rightarrow XW$ ($Z \rightarrow W$)

$XZ \rightarrow W$ (decomposition rule)

$XZ \rightarrow XZ$

$XZ \rightarrow YZ$ ($X \rightarrow Y$)

$XZ \rightarrow Y$ (decomposition rule)

$\rightarrow XZ \rightarrow YW$ (union rule)

Hence True.

b. $\{X \rightarrow Y, XY \rightarrow Z\} \rightarrow X \rightarrow Z$??

$XY \rightarrow Z$

$XX \rightarrow Z$ (pseudotransitivity rule as $X \rightarrow Y$)

$\rightarrow X \rightarrow Z$

Hence True.

c. $\{XY \rightarrow Z, Y \rightarrow W\} \rightarrow XW \rightarrow Z$??

$W \rightarrow W$

$X \rightarrow X$

$Y \rightarrow YW$

$Z \rightarrow Z$

$WX \rightarrow WX$

$WY \rightarrow WY$

$WZ \rightarrow WZ$

$XY \rightarrow WXYZ$

$XZ \rightarrow XZ$

$YZ \rightarrow WYZ$

Therefore $WX \rightarrow Z$ is not true

You can also find the attribute closure for WX and show that closure set does not contain Z .

Question 2:

Consider a relational scheme R with attributes A,B,C,D,F and the FDs

A \rightarrow BC

B \rightarrow E

CD \rightarrow EF

Prove that functional dependency AD \rightarrow F holds in R.

Step 1 : A \rightarrow BC (Given)

Step 2 : A \rightarrow C (Decomposition Rule applied on step 1)

Step 3 : AD \rightarrow CD (Augmentation Rule applied on step 2)

Step 4 : CD \rightarrow EF (Given)

Step 5 : AD \rightarrow EF (transitivity Rule applied on step 3 and 4)

Step 6 : AD \rightarrow F (Decomposition Rule applied on step 5)

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