Inference Rules For Functional Dependencies

Inference Rules For Functional Dependencies -

Let S be the set of functional dependencies that are specified on relation schema R. Numerous other dependencies can be inferred or deduced from the functional dependencies in S.

Example :

Let S = {A ? B, B ? C}

We can infer the following functional dependency from S:

A ? C

Armstrong's Inference Rules -

Let A, B and C and D be arbitrary subsets of the set of attributes of the giver relation R, and let AB be the union of A and B. Then,??

Reflexivity :

If B is subset of A, then A ? B

Augmentation :

If A ? B, then AC ? BC

Transitivity :

If A ? B and B ? C, then A ? C.

Projectivity or Decomposition Rule :

If A ? BC, Then A ? B and A ? C

Proof :

Step 1 : A ? BC (GIVEN)

Step 2 : BC ? B (Using Rule 1, since B ? BC)

Step 3 : A ? B (Using Rule 3, on step 1 and step 2)

Union or Additive Rule :

If A?B, and A?C Then A?BC.

Proof :

Step 1 : A ? B (GIVEN)

Step 2 : A ? C (given)

Step 3 : A ? AB (using Rule 2 on step 1, since AA=A)

Step 4 : AB ? BC (using rule 2 on step 2)

Step 5 : A ? BC (using rule 3 on step 3 and step 4)

Pseudo Transitive Rule :

If A ? B, DB ? C, then DA ? C

Proof :

Step 1 : A ? B (Given)

Step 2 : DB ? C (Given)

Step 3 : DA ? DB (Rule 2 on step 1)

Step 4 : DA ? C (Rule 3 on step 3 and step 2)

These are not commutative as well as associative.

i.e. if X ? Y then

Y ? X x (not possible)

Composition Rule :

If A ? B, and C ? D, then AC ? BD.

Self Determination Rule :

A ? A is a self determination rule.

Question 1:

Prove or disprove the following inference rules for functional dependencies.

Note: Read "?" as implies

a. {X ? Y, Z ? W} ? XZ ? YW ??

b. {X ? Y, XY ? Z} ? X ? Z

c. {XY ? Z, Y ? W} ? XW ? Z

Solution :

Method : Use Armstrong's Axioms or Attribute closure to prove or disprove.

a. {X ? Y, Z ? W} ? XZ ? YW ??

XZ ? XZ

XZ ? XW (Z -> W)

XZ ? W (decomposition rule)

XZ ? XZ

XZ ? YZ (X -> Y)

XZ ? Y (decomposition rule)

? XZ ? YW (union rule)

Hence True.

b. {X ? Y, XY ? Z} ? X ? Z ??

XY?Z

XX ? Z (pseudotransitivity rule as X ? Y)

? X ? Z

Hence True.

c. {XY ? Z, Y ? W} ? XW ? Z ??

W ? W

X ? X

Y ? YW

Z ? Z

WX ? WX

WY ? WY

WZ ? WZ

XY ? WXYZ

XZ ? XZ

YZ ? WYZ

Therefore WX ? Z is not true

You can also find the attribute closure for WX and show that closure set does not contain Z.

Question 2:

Consider a relational scheme R with attributes A,B,C,D,F and the FDs

A ? BC

B ? E

CD ? EF

Prove that functional dependency AD ? F holds in R.

Step 1 : A ? BC (Given)

Step 2 : A ? C (Decomposition Rule applied on step 1)

Step 3 : AD ? CD (Augmentation Rule applied on step 2)

Step 4 : CD ? EF (Given)

Step 5 : AD ? EF (transivity Rule applied on step 3 and 4)

Step 6 : AD ? F (Decomposition Rule applied on step 5)

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